QUIZ 8 SOLUTIONS: LESSON 9
SEPTEMBER 17, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the general solution to

$$\frac{dy}{dt} - e^t y = e^t.$$

Solution: This is a FOLDE and in the correct form. We go through our steps:

Step 1: Find P, Q

$$P(t) = -e^t, \quad Q(t) = e^t$$

Step 2: Find integrating factor

$$\begin{aligned} u(t) &= e^{\int P(t)\, dt} \\ &= e^{\int (-e^t)\, dt} \\ &= e^{-e^t} \end{aligned}$$

Step 3: Set up the Solution

$$y(t)u(t) = \int Q(t)u(t)\, dt$$

$$\Rightarrow \quad y(t)\underbrace{e^{-e^t}}_{u(t)} = \int \underbrace{(e^t)}_{Q(t)}\underbrace{e^{-e^t}}_{u(t)}\, dt \Rightarrow \quad y(t)e^{-e^t} = \int e^t e^{-e^t}\, dt$$

The right hand side is a u-sub problem: let $u = -e^t$, then $du = -e^t\, dt$. Thus,

$$\int e^t e^{-e^t}\, dt = \int -e^u\, du = -e^u + C = -e^{-e^t} + C.$$

Next, we write

$$\begin{aligned} y(t)e^{-e^t} &= -e^{-e^t} + C \\ \Rightarrow \quad y(t) &= \frac{-e^{-e^t} + C}{e^{-e^t}} \\ &= e^{e^t}(-e^{-e^t} + C) \\ &= \boxed{-1 + Ce^{e^t}} \end{aligned}$$

2. [5 pts] Find the general solution to

$$3x^2y + x^3y' = 2\sec^2 x \tan x.$$

Assume that $x > 0$.

Solution: This is a FOLDE but it is not in the correct form. We divide both sides by x^3 to get

$$y' + \frac{3x^2}{x^3}y = \frac{2\sec^2 x \tan x}{x^3}$$

which becomes

$$y' + \frac{3}{x}y = \frac{2\sec^2 x \tan x}{x^3}.$$

Now, that this is in the correct form we may apply our steps.

Step 1: Find P, Q

$$P(x) = \frac{3}{x}, \quad Q(x) = \frac{2\sec^2 x \tan x}{x^3}$$

Step 2: Find the integrating factor

$$\begin{aligned} u(x) &= e^{\int P(x)\,dx} \\ &= e^{\int \frac{3}{x}\,dx} \\ &= e^{3\ln x} \\ &= e^{\ln x^3} \\ &= x^3 \end{aligned}$$

where we have assumed that $x > 0$.

Step 3: Set up the Solution

$$y(x)u(x) = \int Q(x)u(x)\,dx$$

$$\Rightarrow \quad y(x)\underbrace{(x^3)}_{u(x)} = \int \underbrace{\frac{2\sec^2 x \tan x}{x^3}}_{Q(x)} \underbrace{(x^3)}_{u(x)}\,dx$$

$$\Rightarrow \quad y(x)x^3 = \int 2\sec^2 x \tan x\,dx$$

We need to integrate the right hand side. Consider

$$2\sec^2 x \tan x = 2\sec x \sec x \tan x = 2\sec x(\sec x \tan x).$$

Recall that

$$\frac{d}{dx}\sec x = \sec x \tan x.$$

Thus, if $u = \sec x$, then $du = \sec x \tan x$ and we write

$$\begin{aligned}
\int 2\sec^2 x \tan x\,dx &= \int 2\sec x(\sec x \tan x)\,dx \\
&= \int 2u\,du \\
&= u^2 + C \\
&= (\sec x)^2 + C \\
&= \sec^2 x + C
\end{aligned}$$

Therefore, we have

$$y(x)x^3 = \sec^2 x + C$$

$$\Rightarrow \quad y(x) = \boxed{\frac{\sec^2 x + C}{x^3}}$$